Q Multiplication in the Wien-bridge Oscillator

The Wien-bridge oscillator earns its name from the typical bridge arrangement of the feedback loops (fig.1). This configuration is capable of delivering a clean output sine wave using a low-Q frequency-determining R-C network and some negative feedback.

We are interested in computing a figure of merit or Q for the oscillator that will account for harmonic rejection at the output, finding its relationship with the R-C network´s Q.

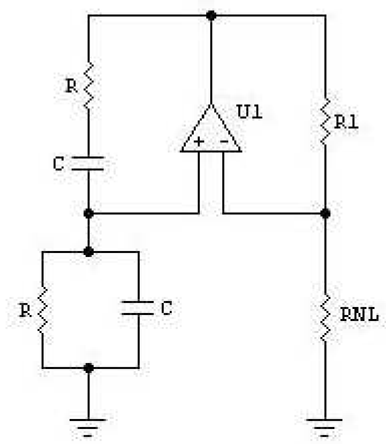

Fig.1 Wien-Bridge Oscillator

We shall start considering a signal V_1 fed back from the output to the amplifier´s inputs and resulting in a differential input signal ($V_+ - V_-$). We may write:

$$\frac{V_+ - V_-}{V_1} = \frac{j\omega RC}{1-\omega^2 R^2 C^2 + 3j\omega RC} - \frac{R_{NL}}{R_1 + R_{NL}}$$

$$= G(j\omega) - K \qquad \ldots(1)$$

Here, it is assumed that the differential amplifier´s input-impedance is very high. We can recognize the R-C frequency-sensitive network as being a 2nd order bandpass filter. This type of filter has a transfer function in the Laplace domain given by:

$$G(s) = \frac{as}{bs^2 + cs + 1}$$

with a, b and c being circuit constants. For steady-state sinusoidal operation the above expression may be written as:

$$G(j\omega) = \frac{aj\omega}{1 - b\omega^2 + jc\omega} \qquad \ldots(2)$$

with s = jω.

The resonant frequency ω_o is given by:

$$\omega_0 = \frac{1}{\sqrt{b}}$$

Gain at resonance is:

$$G(j\omega_0) = \frac{a}{c}$$

The 3dB bandwidth can be shown to be:

$$\Delta\omega = \frac{c}{b}$$

The network´s Q is:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\sqrt{b}}{c} \qquad \ldots(3)$$

Then, eq.(2) may be written as:

$$G(j\omega) = \frac{\dfrac{G(j\omega_0)}{Q\omega_0} j\omega}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2 + j\dfrac{\omega}{\omega_0 Q}} \qquad \ldots(4)$$

The amplitude-frequency response is described by:

$$|G(j\omega)| = \frac{G(j\omega_0)\dfrac{\omega}{\omega_0 Q}}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_0}\right)^2\right]^2 + \left(\dfrac{\omega}{\omega_0 Q}\right)^2}}$$

The phase-angle response may be obtained from eq.(4):

$$\Phi(\omega) = \frac{\pi}{2} - \tan^{-1}\left[\frac{\dfrac{\omega}{\omega_0 Q}}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2}\right]$$

We need now calculate the derivative of $\Phi(\omega)$ with respect to ω. From tables for derivatives we find that:

$$\frac{d}{dx}\tan^{-1}(y) = \frac{1}{1+y^2}\cdot\frac{dy}{dx}$$

Then:

$$\frac{d\Phi(\omega)}{d\omega} = \Phi'(\omega) = -\frac{\left[1+\left(\frac{\omega}{\omega_0}\right)^2\right]\frac{1}{\omega_0 Q}}{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2+\left(\frac{\omega}{\omega_0 Q}\right)^2}$$

Evaluating $\Phi'(\omega)$ at $\omega = \omega_o$:

$$\Phi'(\omega_0) = -\frac{2Q}{\omega_0}$$

or:

$$Q = -\frac{1}{2}\omega_0\Phi'(\omega_0) \qquad \ldots(5)$$

At this point we can verify, using eq.(3), that the Q of the frequency-sensitive network is 1/3. In the next section we will see how a Q multiplication takes place due to bridge operation in the oscillator.

Q Multiplication

Multiplying eq.(1) by A_d yields the condition that must be satisfied for oscillations to take place:

$$[G(j\omega)-K]A_d = 1 \qquad \ldots(6)$$

$G(j\omega)$ is the transfer function of the frequency-sensitive network.
K is the transfer function of the non-linear network.
A_d is the amplifier´s open-loop gain.

At the oscillation´s frequency, $G(j\omega)$ and K must be real if A_d is a real quantity. For ideal OP-AMPS, A_d is considered a real number, actually very large. For real-world devices with internal frequency compensation, A_d is a complex quantity having a low-frequency pole, and its magnitude rolls-off at 20dB per decade above the corner frequency. It may be shown that A_d can be considered to be a real quantity in eq.(6) if:

$$GBW/f_{osc} >> 9$$

where GBW is the gain-bandwidth product of the OP-AMP and f_{osc} is the oscillation´s frequency in hertz.

Selectivity of the frequency-dependent feedback loop is given by its Q [eq.(5)]:

$$Q_1 = -\frac{1}{2}\omega_0 \frac{d\Phi_1}{d\omega}$$

Total selectivity resulting from the action of the two feedback loops may be described by:

$$Q_{12} = -\frac{1}{2}\omega_0 \frac{d\Phi_{12}}{d\omega}$$

For small variations of frequency and phase angles:

$$\frac{Q_1}{Q_{12}} = \frac{\Delta\Phi_1}{\Delta\Phi_{12}}$$

From fig.2.b we may write:

$$|G(j\omega)|\sin\Delta\Phi_1 = |G(j\omega) - K|\sin\Delta\Phi_{12}$$

and for small phase shifts:

$$|G(j\omega)|\Delta\Phi_1 = |G(j\omega) - K|\Delta\Phi_{12}$$

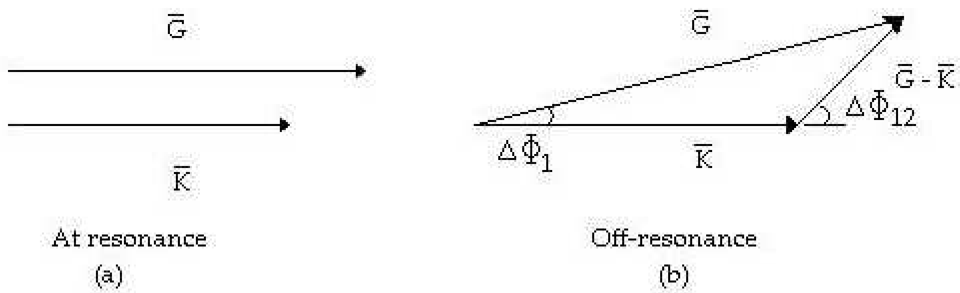

Fig.2 G(jω) and K represented as phasors

We may deduce that:

$$\frac{Q_{12}}{Q_1} = \left|\frac{G(j\omega)}{G(j\omega) - K}\right| \qquad \ldots(7)$$

At the oscillation´s frequency:

$$|G(j\omega) - K| = \frac{1}{A_d}$$

and:

$$G(j\omega) = \frac{1}{3}$$

Then:

$$\frac{Q_{12}}{Q_1} = \frac{A_d}{3} \qquad \ldots(8)$$

Thus, the bridge is very nearly at balance and Q_{12} is many times Q_1.

Typical open-loop voltage gain variation with frequency is indicated in fig.3 for an OP-AMP with internal frequency compensation. Here, G_o is the DC voltage gain expressed in decibels and f_o is the low-frequency pole. G is the voltage gain in decibels at frequency f. f_u is the unity-gain frequency.

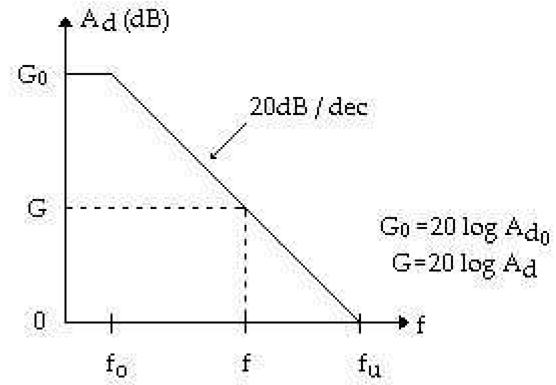

Fig.3 Open-loop frequency response of an internally compensated OP-AMP

The following holds due to the 20dB per decade roll-off:

$$GBW = A_{do} f_o = A_d f = f_u \qquad \ldots(9)$$

At a frequency f, the open-loop voltage gain is:

$$A_d = \frac{GBW}{f} \qquad \ldots(10)$$

Substituting into eq. (8):

$$\frac{Q_{12}}{Q_1} = \frac{GBW}{3f}$$

Then:

$$Q_{12} = \frac{GBW}{9f}$$

The effective Q then varies inversely with frequency.

A typical Q multiplication factor at 1kHz, with a 4MHz gain-bandwidth product OP-AMP is:

$$\frac{Q_{12}}{Q_1} = \frac{4\times 10^6}{3\times 10^3} = 1333.33$$

This would give a value of 444.44 for Q_{12}.

For the case of the modified Wien-bridge oscillator using a single variable resistor for frequency control:

$$G(j\omega) = \frac{jk_1\omega RC}{1 - k_1k_2\omega^2R^2C^2 + j[k_1(k_2+1)+1]\omega RC}$$

$$G(j\omega_0) = \frac{k_1}{k_1(k_2+1)+1} \qquad \ldots(11)$$

$$\omega_0 = \frac{1}{\sqrt{k_1k_2}RC}$$

$$Q_1 = \frac{\sqrt{k_1k_2}}{k_1(k_2+1)+1} \qquad \ldots(12)$$

Eq.(7) yields the Q multiplication factor:

$$\frac{Q_{12}}{Q_1} = A_d\frac{k_1}{k_1(k_2+1)+1} \qquad \ldots(13)$$

Q_{12} is then given by:

$$Q_{12} = A_d\frac{k_1}{k_1(k_2+1)+1}\cdot\frac{\sqrt{k_1k_2}}{k_1(k_2+1)+1}$$

$$= \frac{GBW}{f}\cdot\frac{k_1\sqrt{k_1k_2}}{[k_1(k_2+1)+1]^2}$$

$$= 2\pi \cdot RC \cdot GBW \cdot \frac{k_1^{\,2} k_2}{\left[k_1(k_2+1)+1\right]^2} \qquad \ldots(14)$$

If $k_1(k_2+1) >> 1$:

$$Q_{12} \approx 2\pi \cdot RC \cdot GBW \cdot \frac{k_2}{(k_2+1)^2} \qquad \ldots(15)$$

Q_{12} is then approximately constant over one decade.

Using eq.(14) we may calculate the Q_{12} ratio when k_1 varies between $k_{1min} = 10$ and $k_{1MAX} = 1000$. Then:

$$Q_{12} Ratio = \frac{Q_{12}(k_{1MAX})}{Q_{12}(k_{1\min})} = \left[\frac{k_{1MAX}}{k_{1\min}} \cdot \frac{k_{1\min}(k_2+1)+1}{k_{1MAX}(k_2+1)+1}\right]^2$$

$$= \left[\frac{10(k_2+1)+1}{1000(k_2+1)+1}\right]^2 \times 10^4 \qquad \ldots(16)$$

Table I summarizes Q_{12} Ratio and Q_{12} values as given by eqs. (16) and (15), with k_2 as a parameter, for a Wien-bridge oscillator designed for operation over the 1kHz to 10kHz decade.

TABLE I

k_2	Q_{12} Ratio	Q_{12} (aprox.)
0.25	1.1645	40.47
0.5	1.1362	39.74
1	1.1014	31.61
2	1.0671	19.87
10	1.0181	3.30

Calculations for $k_2 = 2$ have been made with GBW = 4MHz, R = 470 ohms, and C = 7.57nF. For other values of k_2, C has been changed accordingly, so the same 1kHz to 10kHz decade may be tuned.

From the total selectivity point of view, low values for k_2 are preferred. We may also observe that given any frequency decade, selectivity at the lower end is slightly greater than that at the upper end (Q_{12} Ratio>1).

Eq. (15) indicates that higher decades exhibit smaller Q_{12} values (the higher the decade, the smaller the RC product).

Some THD measurements made on the modified Wien-bridge oscillator with $k_2 = 2$ and a 6-Volt peak-amplitude output sine wave are shown below. Measurements were conducted using a 334A Hewlett-Packard Distortion Analyzer.

1kHz to 10kHz decade

THD at:	1kHz	10kHz
is:	0.1%	0.03%

2kHz to 20kHz decade

THD at:	2kHz	20kHz
is:	0.09%	0.06%

200Hz to 2kHz decade

THD at:	200Hz	2kHz
is:	0.4%	0.017%

20Hz to 200Hz decade

THD at:	20Hz	200Hz	
is:	3.8%	0.04%	(using 1 stabilising lamp)
is:	2.47%	0.038%	(using 2 stabilising lamps in series)

Some comments

The lower end of the 20Hz to 200Hz decade is adversely affected by environmental noise and non-linear distortion introduced by the stabilising lamp. Three or four of these lamps should be series-connected in order to reduce THD to acceptable levels. Miniature lamp types should be preferred (they are less bulky). Also, the oscillator should be adequately shielded from external noise sources, such as fluorescent lamps, computers, switch-mode power supplies, etc.

When conducting measurements with the Distortion Analyzer at frequencies above 1kHz, a high-pass filter may be switched-in for noise rejection. This may help lower the THD reading.

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