Analysis of the Tuggle Front End – Part II

We shall now consider the Tuggle tuner delivering power to a load. First, we must account for the parallel RF losses of the unloaded tuned circuits. Let R_P represent the losses of the tank circuit comprising L_1 and C, and R_{TNK} those of L_2 and C_2 (please see Fig. 1).

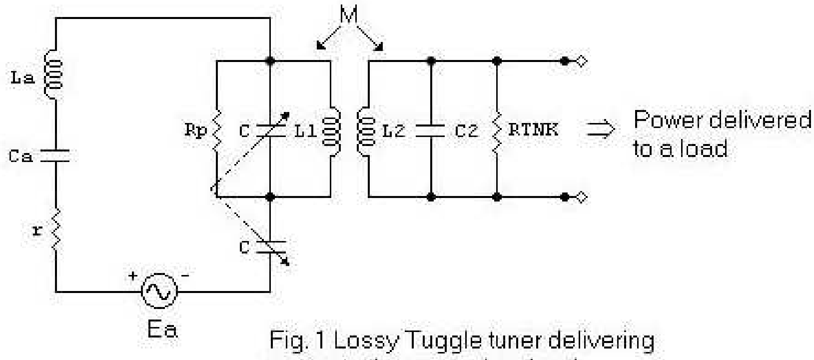

Fig. 1 Lossy Tuggle tuner delivering power to the secondary load

The load consists of a diode detector D in series with an audio load R_L (usually an audio transformer matching a pair of 2k ohms DC resistance magnetic headphones or low-impedance sound powered phones to the detector) and is coupled to the tuner via the magnetic coupling existing between L_1 and L_2, being M the mutual inductance of the coils. The secondary is tuned to the same radian frequency as the primary. An schematic for the load can be seen in Fig. 2.

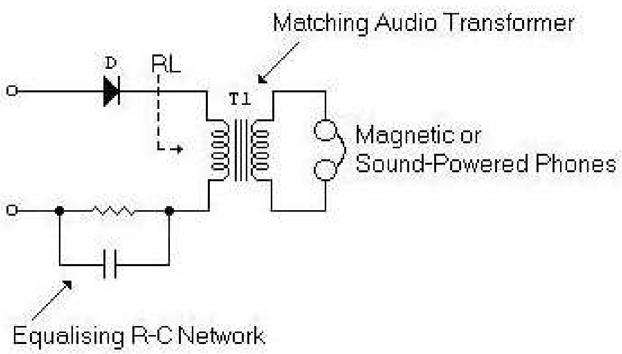

Fig. 2 Load coupled to the Tuggle tuner

Usually, it is assumed that optimum RF power transfer occurs when the antenna-ground system resonance resistance r is matched to the unloaded-secondary resonance parallel resistance R_{TNK}, with the diode detector´s input resistance matched to this combination. Thus,

$$\frac{1}{R_{OPT}} = \frac{1}{R_{TNK}} + \frac{1}{R_{TNK}} + \frac{1}{\left(\frac{R_{TNK}}{2}\right)}$$

or $R_{OPT} = \frac{R_{TNK}}{4}$.

This is the overall parallel RF resistance of the secondary tank under matched conditions, and suggests that the unloaded Q of this tank circuit has been reduced to ¼ of its value.

In this case, the net parallel resistance to be coupled to the primary is:

$$R_2 = \frac{R_{TNK}\left(\frac{R_{TNK}}{2}\right)}{R_{TNK} + \left(\frac{R_{TNK}}{2}\right)}$$

$$= \frac{R_{TNK}}{3} \qquad \ldots(1)$$

We shall work on this later.

<u>Some circuit equivalents</u>

In Fig.1, let´s replace L_1 and the coupled secondary circuit by the equivalent shown in Fig.3.a, which in turn can be replaced by the transformer circuit shown in Fig.3.b.

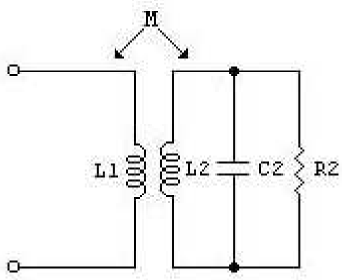

Fig.3.a Magnetically coupled circuits

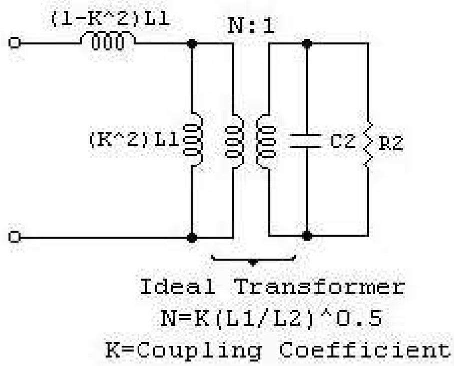

Fig.3.b Equivalent transformer circuit

In the transformer circuit, the impedance coupled to the primary side consists of a capacitance C_2 / N^2 in parallel with a resistance N^2R_2. Both are in parallel with the magnetizing inductance K^2L_1 (please see Fig.4.a).

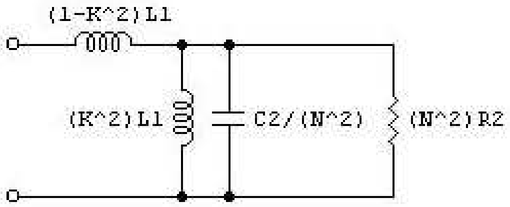

Fig.4.a Equivalent circuit as seen from the primary side

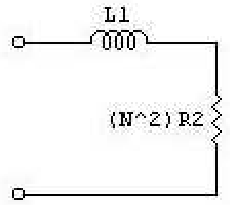

Fig.4.b Reduced equivalent at $\omega = \omega_r$

K^2L_1 and C_2 / N^2 resonate at a frequency

$$\omega_0 = \frac{1}{\sqrt{K^2 L_1\left(\frac{C_2}{N^2}\right)}}$$

$$= \frac{1}{\sqrt{L_2 C_2}} = \omega_r$$

as $N^2 = K^2 \frac{L_1}{L_2}$.

The equivalent circuit of Fig.4.a reduces to that of Fig.4.b, taking into account that for crystal set use, normally K<<1.

Up to this point, in the tuner side we have the equivalent circuit depicted in Fig.5.

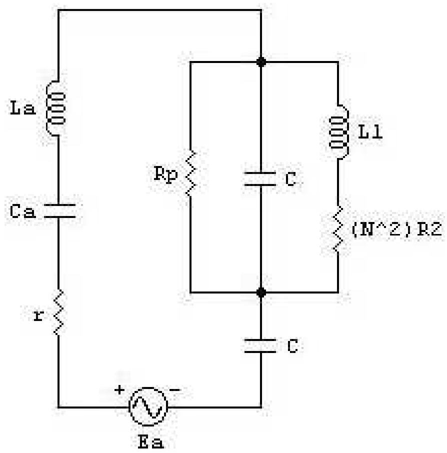

Fig.5 Equivalent circuit of the tuner with coupled secondary load

The series-coupled resistance N^2R_2 can be transformed into a resistance R_{P1} in parallel with R_P. Using the known series-to-parallel "loss resistance" transformation we get:

$$R_{P1} = \frac{(\omega_r L_1)^2}{N^2 R_2} \qquad \ldots(2)$$

Let $Q_2 = \frac{R_2}{\omega_r L_2}$. Then:

$$R_{P1} = \frac{\omega_r L_1}{K^2 Q_2} \qquad \ldots(3)$$

Next, we compute the equivalent resistance R_T of the parallel combination of R_P and R_{P1}. It is given by:

$$R_T = \frac{R_P R_{P1}}{R_P + R_{P1}}$$

Substituting R_{P1} by its equivalent given by eq.(3):

$$R_T = \frac{R_P\left(\frac{\omega_r L_1}{K^2 Q_2}\right)}{R_P + \frac{\omega_r L_1}{K^2 Q_2}}$$

$$= \frac{R_P \omega_r L_1}{\omega_r L_1 + K^2 Q_2 R_P}$$

Letting $Q_1 = \frac{R_P}{\omega_r L_1}$ (unloaded Q of L_1-C tank) we obtain:

$$R_T = \frac{R_P}{1 + K^2 Q_1 Q_2} \qquad \ldots(4)$$

If $R_T >> \omega_r L_{eq} = \frac{\omega_r L_1}{1 - \omega_r^2 L_1 C}$ (please refer to part I of this study) we can redraw the equivalent circuit of the tuner at resonance as indicated by Fig.6.a, and, by virtue of the above inequality, the resonant frequency ω_r will still be given by:

$$\omega_r^2 L_1 C\left(\frac{2Ca + C}{Ca + C}\right) = 1 \qquad \ldots(5)$$

Applying a parallel-to-series transformation, the equivalent circuit of Fig.6.b is obtained.

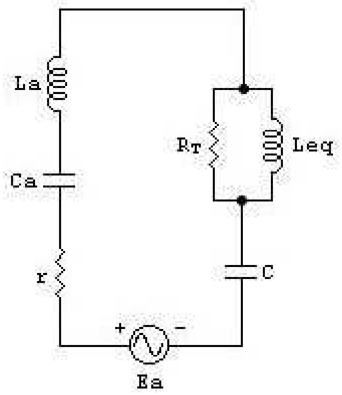

Fig.6.a Equivalent circuit of the tuner when $R_T \gg \omega_r L_{eq}$

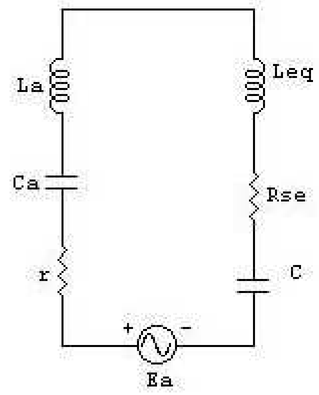

Fig.6.b Equivalent circuit after transformation

The series transformed resistance R_{se} is given by:

$$R_{se} = \frac{(\omega_r L_{eq})^2}{R_T}$$

$$= (\omega_r L_{eq})^2 \left(\frac{1 + K^2 Q_1 Q_2}{R_P} \right)$$

$$= \frac{(\omega_r L_{eq})^2}{R_P} + (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P}$$

Let $R_s = \frac{(\omega_r L_{eq})^2}{R_P}$ and $R_{s1} = (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P}$.

R_s is the series term due to R_P and R_{s1}, that coming from the coupled resistance N^2R_2. Now, maximum RF power transfer to N^2R_2 occurs when:

$$r + R_s = R_{s1}$$

or when:

$$r + \frac{(\omega_r L_{eq})^2}{R_P} = (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P}$$

or

$$\frac{(\omega_r L_{eq})^2}{r} = \frac{R_P}{K^2 Q_1 Q_2 - 1} \qquad \ldots(6)$$

From part I of this study we know that:

$$L_{eq} = \frac{L_1}{1-\omega_r^{\ 2} L_1 C}$$

$$= L_1\left(2+\frac{C}{Ca}\right) \qquad \ldots(7)$$

being

$$\omega_r^{\ 2} = \frac{1}{L_1 C}\left(\frac{Ca+C}{2Ca+C}\right)$$

Then:

$$\left(\omega_r L_{eq}\right)^2 = \frac{1}{L_1 C}\left(\frac{Ca+C}{2Ca+C}\right) L_1^{\ 2}\left(\frac{2Ca+C}{Ca}\right)^2$$

$$= \frac{L_1}{C}\left(2+\frac{C}{Ca}\right)\left(1+\frac{C}{Ca}\right)$$

Eq.(6) is now written as:

$$\frac{L_1}{rC}\left(2+\frac{C}{Ca}\right)\left(1+\frac{C}{Ca}\right) = \frac{R_P}{K^2 Q_1 Q_2 - 1} \qquad \ldots(8)$$

Letting $A = \frac{L_1}{rC}\left(2+\frac{C}{Ca}\right)\left(1+\frac{C}{Ca}\right)$, eq.(8) takes the more compact form:

$$A = \frac{R_P}{K^2 Q_1 Q_2 - 1}$$

Solving for K we obtain:

$$K = \sqrt{\frac{R_P + A}{Q_1 Q_2 A}} \qquad \ldots(9)$$

which gives the value of the coupling coefficient for maximum RF power transfer to the secondary.

Power calculations

The RF power delivered to the secondary load of Fig.1 will be at a maximum at resonance when eq.(6) is satisfied, this is, when r + R_s = R_{s1}. The maximum available power is then:

$$P_{MAX} = \frac{\left(\frac{E_a}{2}\right)^2}{2R_{s1}}$$

$$= \frac{E_a^{\ 2}}{8R_{s1}} \qquad \ldots(10)$$

where E_a is the peak value of the voltage induced in the antenna.

The power delivered to the secondary load R_2 is the same as that dissipated by the coupled resistance N^2R_2. To compute this power we need the voltage across L_1 at resonance. This is the same as the voltage across L_{eq} in Fig.6.a. Then:

$$E_{Leq} = \frac{E_a}{2R_{s1}} j\omega_r L_{eq} \qquad \ldots(11)$$

From Fig.5 we obtain for the voltage across N^2R_2:

$$E_2^{'} = \frac{E_{Leq}}{j\omega_r L_1 + N^2 R_2} N^2 R_2$$

In crystal sets, $N^2 R_2 << \omega_r L_1$, due to the loose coupling between L_1 and L_2. Then:

$$E_2^{'} = \frac{E_{Leq}}{j\omega_r L_1} N^2 R_2$$

$$= \frac{E_{Leq}}{j\omega_r L_1}\left[\frac{(\omega_r L_1)^2}{R_{P1}}\right]$$

$$= \frac{E_{Leq}}{jR_{P1}} \omega_r L_1$$

Bearing in mind eq.(3):

$$E_2^{'} = \frac{E_{Leq}}{j} K^2 Q_2$$

Substituting the value of E_{Leq} given by eq.(11) into the above expression we obtain:

$$E_2^{'} = \frac{E_a}{2R_{s1}} \omega_r L_{eq} K^2 Q_2$$

We can recall that:

$$R_{s1} = (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P} \qquad \ldots(12)$$

Then:

$$\frac{\omega_r L_{eq} K^2 Q_2}{R_{s1}} = \frac{R_P}{\omega_r L_{eq} Q_1}$$

$$= \frac{L_1}{L_{eq}}$$

$$= \frac{Ca}{2Ca + C}$$

Then, we obtain:

$$E_2^{'} = \frac{E_a}{2}\left(\frac{Ca}{2Ca + C}\right)$$

The power dissipated by $N^2 R_2$ is:

$$P_2 = \frac{(E_2^{'})^2}{2N^2 R_2}$$

$$= \frac{\left(\frac{E_a}{2}\right)^2 \left(\frac{Ca}{2Ca + C}\right)^2}{2N^2 R_2} \qquad \ldots(13)$$

Eq.(12) can be written as follows:

$$R_{s1} = (\omega_r L_{eq})^2 \left(\frac{N^2 L_2}{L_1}\right)\left(\frac{Q_1 Q_2}{R_P}\right)$$

$$= (\omega_r L_{eq})^2 \left(\frac{N^2 L_2}{L_1}\right)\left(\frac{1}{\omega_r L_1}\right)\left(\frac{R_2}{\omega_r L_2}\right)$$

$$= \left(\frac{L_{eq}}{L_1}\right)^2 N^2 R_2$$

$$= \left(\frac{2Ca + C}{Ca} \right)^2 N^2 R_2$$

or:

$$N^2 R_2 = \left(\frac{Ca}{2Ca + C} \right)^2 R_{s1}$$

Substituting this equivalence into eq.(13):

$$P_2 = \frac{E_a^{\;2}}{8R_{s1}} = P_{MAX}$$

according to eq.(10).

P_{MAX} is then dissipated by N^2R_2 and by consequence, this power is delivered to the secondary load.

Some experimental results

Two coils, L_1 and L_2, were wound on 4.5" diameter styrene forms using 660/46 Litz wire. L_1 measured 152 uH and L_2, 222 uH. A two-gang 475 pF variable capacitor with bakelite insulation was used to tune L_1. L_2 was tuned with a 480 pF variable capacitor with ceramic insulators.

Unloaded Qs for each of the tuned circuits were measured at three frequencies. Accordingly, the corresponding RF losses were calculated. Data is tabulated below.

f	Q_1	Q_{2UL}	Q_2	R_{Pcalc}	$R_{TNKcalc}$	C	C_2
530kHz	700	810	270	354.32 kohms	598.816 kohms	450 pF	406 pF
1 MHz	585	630	210	558.70 kohms	878.766 kohms	100 pF	114 pF
1.7 MHz	300	318	106	487.072 kohms	754.065 kohms	31 pF	39.5 pF

C : two-gang 475 pF variable capacitor with bakelite insulation
C_2 : 480 pF variable capacitor with ceramic insulation
Q_{2UL} : unloaded Q of L_2-C_2 combination
Q_1, Q_2 : defined in the text
R_P, R_{TNK}: defined in the text

Using the tabulated data, values for the optimum coupling coefficient K will be calculated for a working crystal set.

f = 530 kHz

L_1 = 152 uH — Ca = 200 pF (assumed) — r = 30 ohms (assumed)
A = 1.555 x 10^5 ohms — Q_1Q_2A = 2.938 x 10^{10} ohms — K = 4.165 x 10^{-3}

Check: $\frac{R_P}{1 + K^2 Q_1 Q_2} = 82.8kohms >> \omega_r L_{eq} = 2.214kohms$

f = 1 MHz

L_1 = 152 uH — Ca = 200 pF (assumed) — r = 30 ohms (assumed)
A = 1.9 x 10^5 ohms — Q_1Q_2A = 2.334 x 10^{10} ohms — K = 5.663 x 10^{-3}

Check: $\frac{R_P}{1+K^2Q_1Q_2} = 113.1 kohms >> \omega_r L_{eq} = 2.387 kohms$

f = 1.7 MHz

L_1 = 152 uH Ca = 200 pF (assumed) r = 30 ohms (assumed)

A = 4.068 x 10^5 ohms Q_1Q_2A = 1.29 x 10^{10} ohms K = 8.324 x 10^{-3}

Check: $\frac{R_P}{1+K^2Q_1Q_2} = 152 kohms >> \omega_r L_{eq} = 3.498 kohms$

Comments

The values obtained for the coupling coefficient K hold for $R_2 = \frac{R_{TNK}}{3}$, as discussed previously. The transformed antenna-ground system resonance resistance, as seen from the secondary, will be equal to R_2, or $\frac{R_{TNK}}{3}$, as we are dealing with maximum power transfer to R_2. Under these conditions, the loaded Q of the secondary circuit will be 1/6 of the unloaded value. For other load conditions, the respective data should be entered into eq.(9).

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